Integration answers and solutions.

1.

a) $x^8/8 + C$

b) $(5/4)x^4 + C$

c) $x^6/2 + 7x^{11}/11 + C$

d) $u = x^3$, $du = 3x^2 dx$, so the integral becomes $\int (u + 10) du = u^2/2 + 10u + C$,

that is $x^6/2 + 10x^3 + C$ (after going back to the original variable x).

Alternatively you can expand and integrate term by term.

e) Expand and integrate term by term.

f) $u = x^2 = 3$, $du = 2x dx$, so the integral becomes
$\int u^{-2} du = -1/u + C = -1/(u^2 + 3) + C$

g) $u = x^3 + 2$, $du = 3x^2 dx$, so the integral becomes $\int (1/3) u^{1/2} = (2/9) u^{3/2} + C = (2/9)(x^3 + 2)^{3/2} + C$

2. The volume of the cone of height h and base area A is $V = Ah/3$

in our problem $A = ah^2$, so $V = ah^3/3$. The time derivative $V' = ah^2 h'$

and finally $h' = V'/(ah^2) = V'/A = 50/100 = 1/2$ inches per second.

3. The volume of the balloon is $V = (4/3)\pi r^3$, its surface area is $A = 4\pi r^2$,

so $V' = 4\pi r^2 r'$ and $A' = 8\pi r r' = 2V'/r = 10/10 = 1$