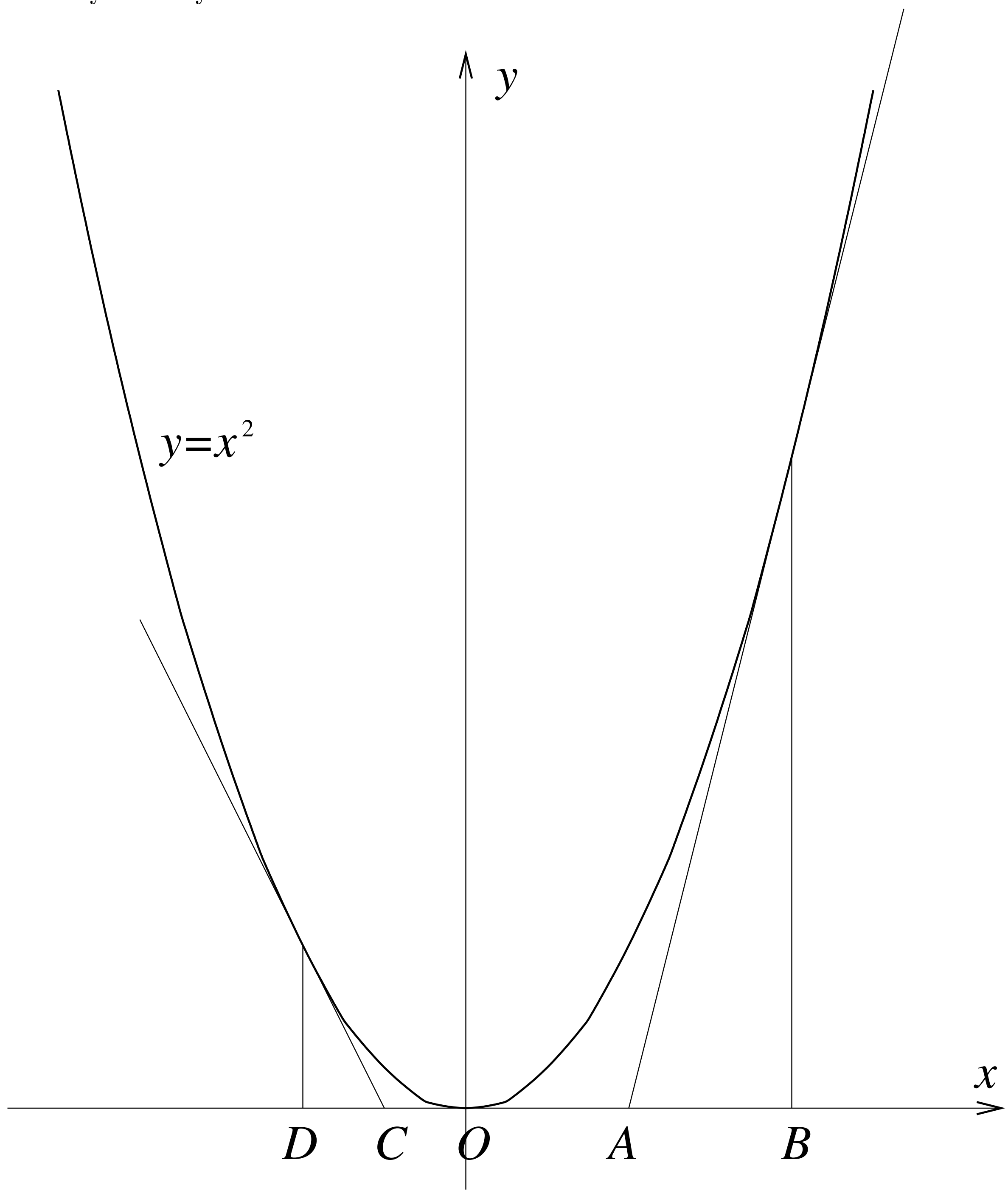
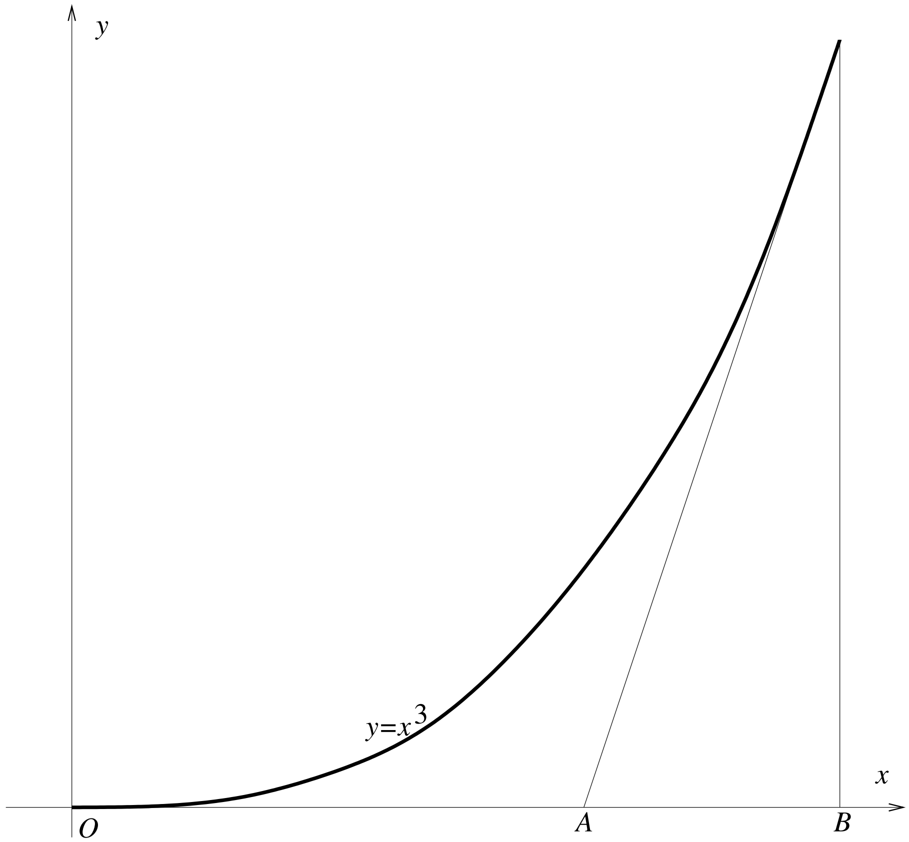
1 Areas, Sums and Tangents

1.1 Hints

Problem 1.1. Does it look like C is exactly half way between D and O and A is exactly half way between O and B?

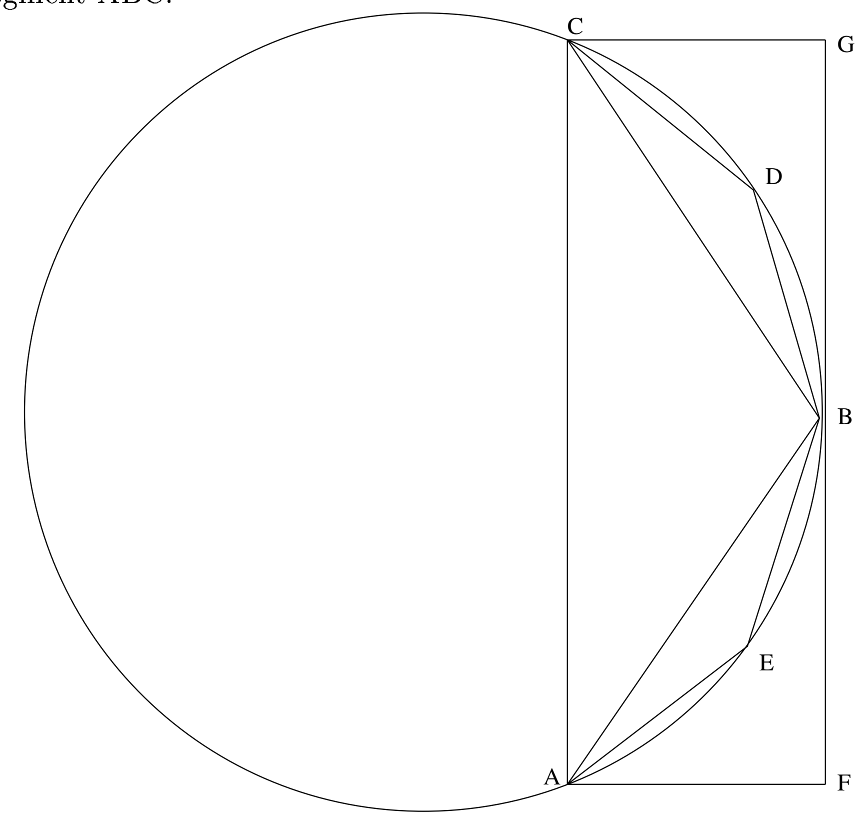


Problem 1.2. Does it look like A is $1/3$ way between B and O?



Problem 1.3. The area of a parabolic segment (after Archimedes)

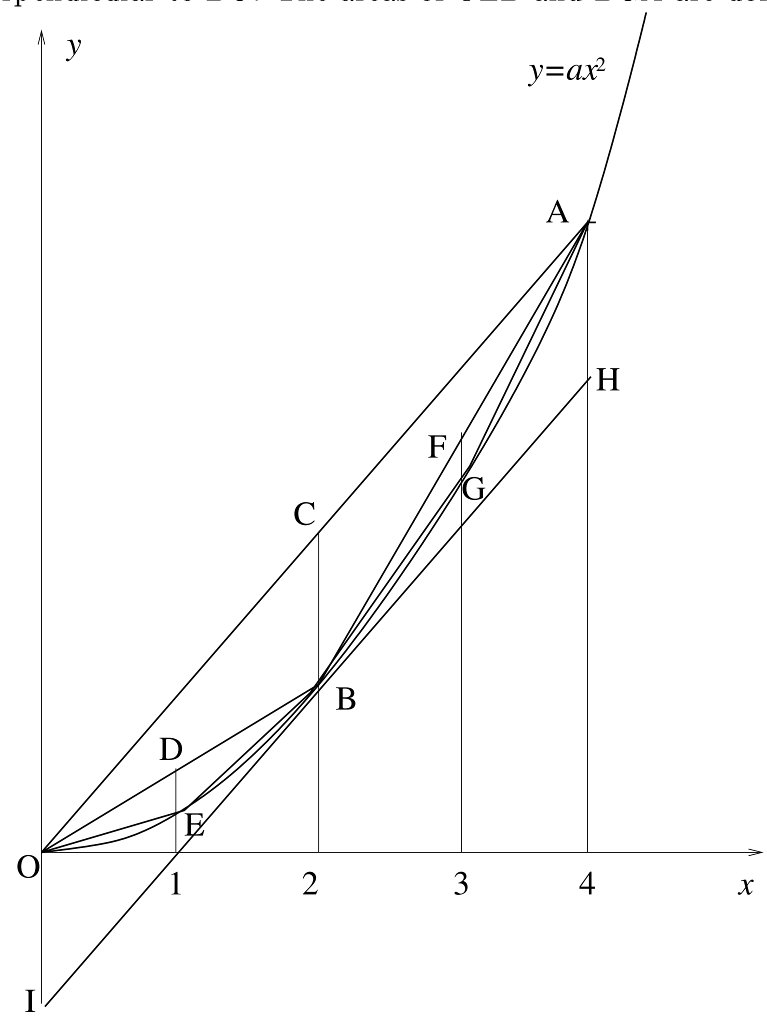
a) The area of the triangle ABC is half the area of the rectangle AFGC, therefore the area of the triangle ABC is more than half the area of the segment ABC.



It follows that the area of the segment AEB +the area of the segment BDC is less than half the area of the segment ABC.

b) The vertical distance d between the parabola $y = ax^2$ and the segment of a straight line OA is the quadratic function with the leading coefficient $-a$ that vanishes at $x = 0$ and $x = 4$, so $d(x) = ax(4 - x)$. Where is its maximum? The answer to this question will indicate why the tangent to the parabola at point B is parallel to OA. It also will follow that ABO has the biggest area among all the triangles inscribed into the segment ABO.

Another approach is to use problem 1 to calculate the sloope of the tangent. Notice that the segment BC cuts triangle ABO into 2 triangles, with the common base BC. What are the heights of these triangles which are perpendicular to BC? The areas of OEB and BGA are done similarly.



Look at the parallelogram IOAH and compare the area of the triangle OBA and the area of the parabolic segment OBA.

I will discuss the finer points of the rest of this problem set in the class, it looks like the computations for these are rather straight forward, apart from the formulas for the sum $1^k + 2^k + ... + N^k$ and the sum of the geometric series, you can look these up on the net.