2 Differentiation and Its Rules

Problem 2.1. Differentiate $\sqrt{x}$ by simplifying *the difference quotient*

$$\frac{\sqrt{x}-\sqrt{a}}{x-a}$$

so you could evaluate it at $x = a$. Do the same for $\sqrt[3]{x}$, $\sqrt[4]{x}$, $\sqrt[5]{x}$ etc. Try to generalize to $\sqrt[n]{x}$. Does it look like problem 1.6 "upside down?" Do you see any connection to the *power rule* $(x^k)' = kx^{k-1}$?

Problem 2.2. Explain why all the polynomials are differentiable, or, to put it differently, why you can always evaluate $(p(x) - p(a))/(x - a)$ for $x = a$ when $p(x)$ is a polynomial. See section 1.2 of my lecture notes that explains division of polynomials.

Problem 2.3. Differentiate $\sqrt{x} + \sqrt[3]{x}$ and $20\sqrt{x}$.

Problem 2.4. Suppose that you can differentiate each of the functions $f(x)$ and $g(x)$, i.e. you can evaluate

$$\frac{f(x)-f(a)}{x-a} \text{ and } \frac{g(x)-g(a)}{x-a}$$

for $x = a$. Can you write a formula for $(f + g)'$ in terms of f' and g'? It is called *Sums Rule*. Now get a formula for $(cf)'$ where c is a constant. It's called *Multiplier Rule*. These two rules make defferentiation of polynomials easy. See section 1.1 of my lecture notes for a general discussion of functions.

Problem 2.5. Express $(fg)'$ in terms of f, f', g and g'. You can try to guess the formula by examining the case when f and g are different powers of x. The answer to this problem is called *Leibniz* or *Product Rule.*

Problem 2.6. Can you think of a clever way to differentiate $(1 + x^2)^{10}$ without expanding the 10th power of the parenthesis? Newton came up with a general formula that makes calculating $f(g(x))'$ easy. It is called *Chain Rule.* Again you can try to guess it by considering some examples.

See section 2.2 of my lecture notes for a discussion of differentiation rules and some additional exercises. Click on "More links related to math" to pick some homework problems from Spring 2003 HSSP.